Towards a Conceptual Model for Quantum Mechanics

Carlton Frederick*

Central Research Group, 127 Pine Tree Road, Ithaca, NY 14850

Abstract

With the decline of the Copenhagen interpretation of quantum mechanics and the recent experiments indicating that quantum mechanics *does* actually embody 'objective reality', one might ask if a 'mechanical', conceptual model for quantum mechanics could be found. Such a model might not be *the* theory, but might provide a way of thinking about quantum mechanics. We propose such a model.

A previous paper [1] noted that space-time vacuum energy fluctuations implied mass fluctuations and, through general relativity, curvature fluctuations. And those fluctuations are indicated by fluctuations of the metric tensor. The metric tensor fluctuations, there presumed to be described by stochastic variables in the tensor elements, can 'explain' the uncertainty relations and non-commuting properties of conjugate variables. It also argues that the probability density $\Psi^*\Psi$ is proportional to the square root of minus the determinant of the metric tensor (the differential volume element) $\sqrt{-\left\|g_{\mu\nu}\right\|}$.

The present paper extends those ideas to a workable model by arguing that the metric elements are actually not stochastic but are oscillating at a sufficiently high frequency that measured values of same appear stochastic (i.e. crypto-stochastic). This was required to allow that the position probability density $\sqrt{-\left\|g_{\mu\nu}\right\|}$ be a non-stochastic variable. In particular, the oscillations at the position of particles are described as torsional vibrations. A crypto-stochastic oscillating metric yields, among other things, a model of superposition, photon polarization, and entanglement, and all within the confines of a 4-dimensional space-time. Further, this implies the deBroglie view (as opposed to the Copenhagen interpretation) that the particle and wave are different entities. The proposed model is one of 'objective reality' but, of course, as required by Bell's theorem, at the expense of temporal locality.

---

*Electronic address: carl@frithrik.com; URL: http://www.frithrik.com

I. INTRODUCTION

'Weak measurement' experiments [15–17], building on the pioneering work of Yakir. Aharonov and Lev Vaidman [18], have dealt a serious if not mortal blow to the Copenhagen interpretation of quantum mechanics, and has given new life to the DeBroglie-Bohm Pilot Wave idea, and a re-emergence of objective reality in quantum mechanics[12–14] [Abandonment of objective-reality says that the physical situation is established by measurement (e.g. the cat is both alive and dead until measured).] In the Pilot wave theory however, the nature and source of those waves has not been explained. The model we propose attempts to explain pilot waves and also develop a way of interpreting a wide range of quantum phenomena. It covers a lot of territory but, by necessity, far from thoroughly. The idea is to build a conceptual scaffolding for a full theory.

At the most primitive level, we regard quantum mechanics as describing stochastic phenomena, in particular the vacuum energy fluctuations throughout space-time, and also with regard to the paths particles take in the space-time. Since vacuum energy fluctuations imply (via general relativity) mass fluctuations. Mass produces curvature which is reflected in fluctuations of the metric tensor components. To first approximation, we take the metric fluctuations to be stochastic. Spaces with stochastic metrics have been investigated by Schweizer [2] for metric spaces and March [3], [4] for Minkowski space. Blokhintsev [5] has considered the physics of a space-time with a small stochastic component.

The following three parts represent a progression of the model. Each successive part is less stochastic than the previous. Deductions of quantum mechanics structure are explored in each part.

Our attempt in Part I is to show that some of the fundamentals of quantum mechanics can be deduced by both imposing apparent stochasticity on the metric tensor and also assuming, and attempting to justify, a few theoretical postulates.

In Part II, we argue that the metric tensor components *cannot* be stochastic, but only *seem* stochastic and are actually oscillations at an immeasurably high frequency. Yet more quantum effects can thereby be explained, including polarization and entanglement. Indeed, Masreliez [6] has derived the Schrödinger equation by imposing a type of oscillation on the metric. And in Part III, we further reduce the indeterminacy by replacing the truly indeterminate stochasticity with chaos. The first order effects are similar for the two cases,

but chaos, while seeming stochastic, is actually a fully deterministic phenomenon.

Whether our model is the 'true' explanation of quantum mechanics is of course debatable. The aim here though, is simply to provide a conceptual model of quantum mechanics that can be comprehended within the usual constraints of logic.

Part I

Stochastic Space-time

II. POSTULATES

1: A Generalization of Mach's Principle

1.1 In the absence of mass, space becomes not flat, but stochastic.

This is the prime reason for uncertainty in quantum mechanics in this model, and the stochasticity directs particle trajectories, the probability of which is given by the wave function, Ψ. In Part III, we'll argue that space-time is not stochastic, but chaotic. The first order effects would be the same, but the philosophy is very different. Chaos is not stochastic but indeterminate yet deterministic.

1.2 The stochasticity is manifested by fluctuations of the metric tensor.

1.3 The mass distribution determines not only the space-time geometry, but also the space-time (apparent) stochasticity.

This and 1.4 encapsulates Mach's Principle.

1.4 The more mass in the space-time, the less the space-time appears stochastic.

1.5 At the position of a mass, the space-time does not fluctuate.

We posit this so that masses aren't pulled apart by the metric fluctuations.

2: The Contravariant nature of Measurements

2.1 All measurements of dynamical variables correspond to *contravariant* components of tensors.

We'll attempt to justify this in Section III

3: The Probability Density P(x,t) Identification at a Venue.

(Note: we use the term space-time 'venue' instead of space-time event [a point in x,y,z,t] to indicate that space-time is 'grainy', i.e. there is a minimum length and time interval. We

assume this to avoid having the infinite vacuum energy fluctuation that would be predicted at a point.)

3.1 P(x,t) is proportional to $\sqrt{-\|g_{\mu\nu}\|}$, i.e. the square root of minus the determinant of the metric tensor.

In differential geometry, the quantity $\sqrt{-\|g_{\mu\nu}\|}dx^1 dx^2 dx^3 dx^4$ corresponds to the Euclidean differential volume element $-dx * dy * dz * dt$. For a particle traveling space-time, we assume then, that the probability of the particle being in a particular differential volume element is proportional to the relative 'size' of the volume element. Note that here we assume the deBroglie idea that the particle actually *is* in a particular location and the wave function is (as deBroglie puts it) the 'ghost wave' that guides the particle.

There is a major constraint on the model: While the metric tensor elements have a stochastic component, if it is to be associated with a probability density, the determinant of the metric tensor, while allowed to change with time, must not be stochastic. That is to say that the probability density is a well-defined (deterministic) quantity in quantum mechanics. This constraint is addressed in Parts II and III.

4: The Wave Function Ψ Identification at a Venue

4.1 There exists a local complex coordinate system where the metric tensor is (at a given venue) diagonal and a component of the metric is the wave function Ψ.

This isn't central to the model but exists simply as an expression of the idea that at present, there are two separate concepts: the metric $g_{\mu\nu}$, and the wave function, Ψ. It is an aim of our geometrical approach to express one of these quantities in terms of the other. We'll address this in Section V.

Incidentally, we also suggest that there is one concept that should be two: waves from the wave equation. One concept is the wave as an indicator of probability, and the other concept as the wave identified with the momentum of the particle. E.g. a wavelength of light and the wave indicating the probability of the photon being at a particular location are two different things. We'll (partially) address this in Part II.

5: Metric Superposition

5.1 If at he position of a particle, the metric due to a specific physical situation is $g_{\mu\nu}(1)$ and the metric due to a different physical situation is $g_{\mu\nu}(2)$, then the metric due to both of the physical situations is $g_{\mu\nu}(3) = \frac{1}{2}[g_{\mu\nu}(1) + g_{\mu\nu}(2)]$.

This is *linear* superposition but for general relativity, this is clearly false as the field

equations are non-linear in terms of the metric tensor. But particle masses are very small (in general relativity terms) and the particle velocities we consider (where the mass is greater than zero) are low compared to the speed of light. So we feel justified in using the *linearized* general relativity field equations (especially as the electro-weak force is some 10^{39} stronger than the gravitational force). The superposition postulate then, is only an approximation, albeit a very good approximation as, for quantum mechanical masses, the linearized field equations diverge only very slightly from the full field equations.

We regard conventional quantum theory as an extension of Newtonian mechanics, and hence it is a linear theory. Our model, as it is an extension of relativity theory, implies a non-linear theory and we would expect superposition to break down at very high particle energies.

III. THE CONTRAVARIANT NATURE OF MEASUREMENTS

The contention is that whenever a measurement can be reduced to a displacement in a coordinate system, it will be represented by contravariant components in the coordinate system. Of course, if the metric tensor $g_{\mu\nu}$ is known, one can calculate covariant quantities from their contravariant counterparts. In our model though, the quantum fluctuations in the vacuum energy is reflected in a stochastic component of the metric elements. So, if we try to use the metric tensor to lower the index of a contravariant quantity, the cooresponding covariant quantity will be at least partially stochastic.

We will attempt to show that, at least for Minkowski space, measurements are contravariant. We'll do that by considering an idealized measurement. Before we do, however, consider as an example the case of measuring the distance to a Schwartzschild singularity (i.e. a black hole) in the Galaxy. Let the astronomical distance to the object be $\ddot{r} \equiv (\overline{\xi}^1)$. The covariant equivalent of the radial coordinate r is ξ_1, and

$$\xi_1 = g_{1\nu}\xi^\nu = g_{11}\xi^1 = \frac{r}{1 - 2Gm/r}$$

so that the contravariant distance to the object is

$$distance = \int_0^{\overline{r}} d\overline{r} = \overline{r}$$
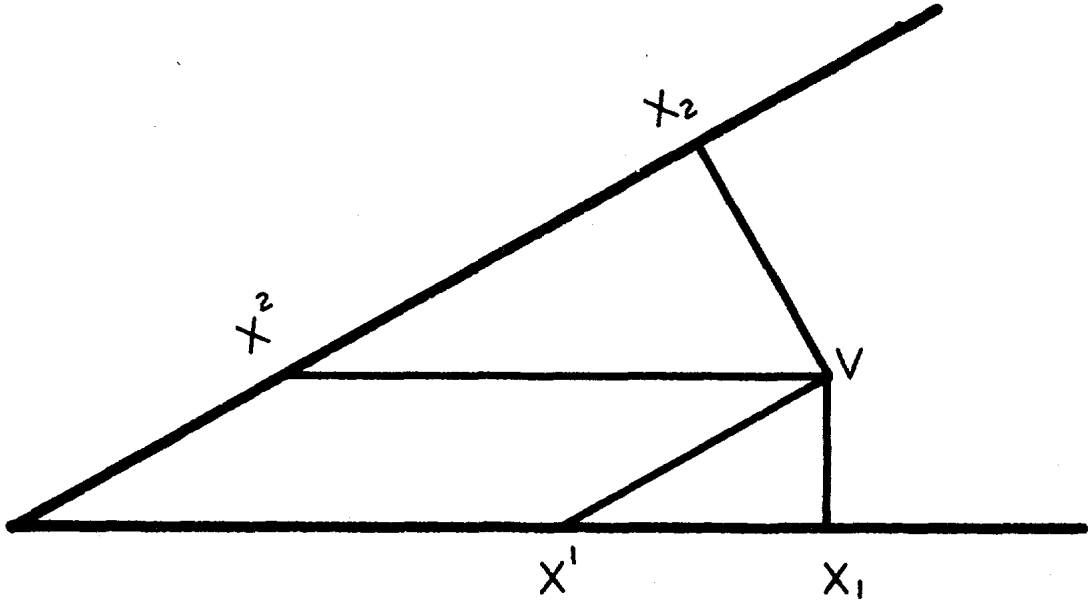
whereas the covariant distance is

5

Fig. 1. Covariant and contravariant components in oblique coordinates

$$\overline{\xi} = \int_0^{\overline{r}} d\left(\frac{r}{1 - 2Gm/r}\right) = \infty$$

From this, it is clear that only the contravariant distance is measurable.

Now as to the postulate, first consider figure 1 showing vector components in a flat space with an oblique 2-dimensional coordinate system. The contravariant coordinates of a point V are given by the parallelogram law of vector addition, while the covariant components are obtained by orthogonal projection onto the axes [7].

We shall now consider an idealized measurement in special relativity, i.e., Minkowski space. Consider the space-time diagram of Fig. 2. We are given that in the coordinate system x', t', an object (the line $m.n$) is at rest.

If one considers the situation from a coordinate system x, t traveling with velocity$v$ along the x'axis, one has the usual Minkowski diagram[8]with coordinate axes $Ox$and Ot and velocity $v = tan\ \alpha$ (where the units are chosen such that the speed of light is unity). OC is part of the light cone.

Noting that the unprimed system is a suitable coordinate system in which to work, we now drop from consideration the original x', t' coordinates.

We wish to determine the 'length' of the object in the x, t coordinate system. At time
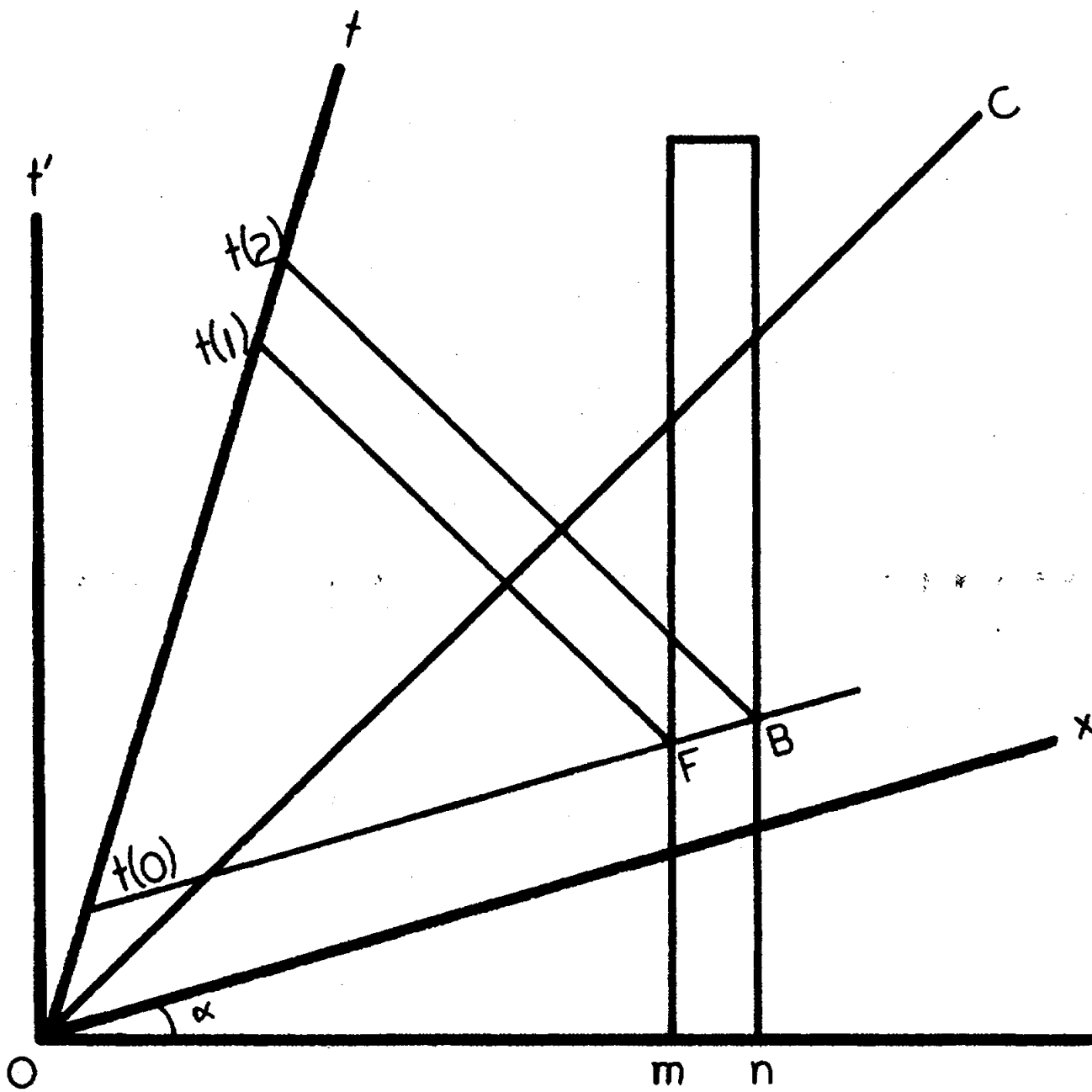
6

Fig. 2. An idealized measurement.

$t(0)$, let a photon be emitted from each end of the object (i.e., from points F and B). The emitted photons will intercept the t axis at times $t(1)$ and $t(2)$. We then can then deduce that the length of the object is $t(2) - t(1)$ (where c=1). The question is: What increment on the x axis corresponds to the time interval $t(2) - t(1)$?

Note that the arrangement that the photons be emitted at time $t(0)$ is nontrivial, but

Fig. 3. Analysis of the idealized measurement.

that it can be done in principle. For the present, let us simply assume that there is a person on the object who knows special relativity and knows how fast the object is moving with respect to the coordinate system. This person then calculates when to emit the photons so that they will be emitted simultaneously with respect to the x, t coordinate system.

Consider now Fig. 3, representing the an analysis of the measurement.

The figure is Fig.2 with the addition of the contravariant coordinates of F and B, x^1 and x^2 respectively. It is easily shown that $t(2) - t(1) = x^2 - x^1$. This is seen by noticing that

8

$x^2 - x^1$ equals the line segment B, F, and that triangle $t(2), t(0), Z$ is congruent to triangle B, $t(0), Z$. If we consider the covariant components, we notice that $x_2 - x_1 = x^2 - x^1$. This is not surprising since coordinate differences (such as $x_2 - x_1$) behave, in flat space, as contravariant objects[9]. To address our postulate, we must consider, not coordinate differences which automatically satisfy the conjecture, but the coordinates individually. Consider in Fig 3. a measurement not of the length of the object, but the position of the trailing edge m of the object m, n. Assume again that at time $t(0)$, a photon is emitted at F and is received at $t(1)$. The observer could then determine the position of m at $t(0)$ by simply measuring the distance $t(1) - t(0)$ on the x axis. Notice that this is the same as the contravariant coordinate value x^1. To determine the corresponding covariant value, here indicated as t_0, one would need to know the angle α (which is determined by the metric tensor).

The metric tensor g_{ik} is defined as $\hat{e}_i \cdot \hat{e}_k$ where $\hat{e}_i$ and $\hat{e}_k$ are the unit vectors in the directions of the coordinate axes x^i and x^k. Therefore in order to consider an uncertain metric (in this 2-space), we can simply consider that the angle α is uncertain. In this case, measurement x^1 is still well-defined ($x^1 = t(1) - t(0)$), but there is no way to determine x_1 because it is a function of the angle α. In this case then, only the contravariant components of position are measurable. It is easy to see from the geometry, that if one were to use the covariant representation of $t(0)$, t_0, one could not obtain a metric-free position measure of m.

The above, of course, can't be considered a rigorous proof of the conjecture that dynamical measurements are only of contravariant quantities, but it is, I believe, strongly suggestive.

IV.   BASIC PHYSICS RESULTS FROM PART I

We derive first the motion of a test particle in the space-time far from other masses. Overall, the space-time is assumed to have enough mass to make the space-time, in the large and on the average, Minkowskian. The requirement that the test particle be far from other masses is so that we can consider that the space time points (venues) in the region can be considered indistinguishable.

Consider a space-time particular venue Θ_1. Let the metric tensor at Θ_1 be $\tilde{g}_{\mu\nu}$ (a tilda over a symbol indicates that it is stochastic). Since $\tilde{g}_{\mu\nu}$ is stochastic, the metric components, by definition, don't have well-defined values. So we cannot know $\tilde{g}_{\mu\nu}$ but we can ask for $P(g_{\mu\nu})$ which is the probability of a particular metric $g_{\mu\nu}$. Note then that since we've ar-

ranged that for close together venues, they and their metric tensors are indistinguishable we have $P_{\Theta 1}(g_{\mu\nu}) = P_{\Theta 2}(g_{\mu\nu})$ where $\Theta 1$ and $\Theta 2$ are two close together venues. $P_{\Theta n}(g_{\mu\nu})$ is to be interpreted as the probability of metric $g_{\mu\nu}$ at venue Θn.

If one inserts a test particle into the space-time, with small position and momentum uncertainties, the particle (probability) motion is given by the Euler-Lagrange equations,

$$\ddot{x}^i + \left\{{i \atop jk}\right\} \dot{x}^j \dot{x}^k = 0,$$

where

$\left\{{i \atop jk}\right\}$ are the Christoffel symbols of the second kind, and where $\dot{x}^j \equiv dx^j/ds$ where s can be either proper time or any single geodesic parameter. Since $\tilde{g}_{\mu\nu}$ is stochastic, these equations generate not a path, but an infinite collection of paths, each with a distinct probability of occurrence. That is to say that $\left\{{i \atop jk}\right\}$ is stochastic (i.e. $\left\{{\tilde{i} \atop jk}\right\}$). Note that because of the stochasticity of the metric tensor, a particle initially at rest is unlikely to stay at rest..

in the absence of near by masses, the test particle motion is easily soluble. Let the particle initially be at (space) venue Θ_0. After time dt, the Euler Lagrange equations yield a distribution of position $D_1(x)$ where $D_1(x)$ represents the probability of the particle being in the region bounded by x and $x + dx$. After another interval dt, the resulting distribution is $D_{1+2}(x)$. From probability theory, this is the convolution,

$$D_{1+2}(x) = \int_{-\infty}^{\infty} D_1(y) D_1(x - y) dy.$$

In this case, $D_1(x) = D_2(x)$. This is so because, since the test particle is far from other masses, the Euler-Lagrange equation will give the same distribution $D_1(x)$ regardless of at which point one propagates the solution. That is to say that $g_{\mu\nu}(x_1)$, $g_{\mu\nu}(x_2)$, $g_{\mu\nu}(x_3)$....are identically distributed random matrices. Thus $D_1(x)$, $D_2(x)$, $D_3(x)$.... are identically distributed random variables. The motion of the test particle is the repeated convolution $D_{1+2+3....n.....}(x)$, which by the central limit theorem is a normal distribution. Thus the position (probability) spread of the test particle at any time $T > 0$ is a Gausssian. The spreading velocity is found to be a constant because of the following: After N convolutions (N large), one obtains a normal distribution with variance σ^2 which, again by the central limit theorem, is N times the variance of $D_1(x)$. Call the variance of $D_1(x)$, a. (i.e. $var(D_1) = a$. The distribution D_1 is obtained after time dt. After N convolutions then,

$$\Delta x = Var\left(D_{\sum_0^n i}\right) = Na.$$

This is obtained after N time intervals dt. One then has,

$$\frac{\Delta x}{\Delta t} = \frac{Na}{N},$$

which is to say that the initially localized test particle probability spreads with a constant velocity a. This is an expression of the spread of the wave function for a free particle.

In the preceding, we've made use of various equations relating to dynamics. So it might be appropriate to say what equations mean in a stochastic space-time.

Since in our model the actual venues (points) of the space-time have a stochastic nature, these venues cannot be used as a basis for a coordinate system nor can derivatives be formed. However, the space-time of common experience (i.e. the laboratory frame) is non-stochastic in the large. It is only in the micro world that the stochasticity is manifested. One can then take this large-scale nonstochastic space-time and mathematically continue it into the micro region. This mathematical construct provides a nonstochastic space to which the stochastic physical space can be referred. The (physical) stochastic coordinates $\tilde{x}^i$ then are stochastic only in that the equations transforming from the laboratory coordinates x^i to the physical coordinates $\tilde{x}^i$ are stochastic.

We'll now use the contravariant observable postulate to derive the uncertainty relation for position and momentum. Similar arguments can be used to derive the uncertainty relations for other pairs of conjugate variables. It will also be shown that there is an isomorphism between a variable and its conjugate, and covariant and contravariant tensors.

We assume that we're able to define a Lagrangian, L. One defines a pair of conjugate variables in the usual way,

$$p_j = \frac{\partial L}{\partial \tilde{q}^j}.$$

Note that this defines p_j a covariant quantity. So that a pair of conjugate variables so defined contains a covariant and a contravariant member (e.g. p_j and q^j). But since p_j is covariant, it (because of the contravariant observable postulate) is not observable in the laboratory frame. The observable quantity is just,

$$\tilde{p}^j = \tilde{g}^{j\nu} p_\nu.$$

But $\tilde{g}^{j\nu}$ is stochastic then so too will be $\tilde{p}^j$. Thus if one member of an observable conjugate variable pair is well defined, the other member is stochastic. To derive an uncertainty relation for a conjugate variable pair, consider the following,

$$\triangle q^1 \triangle p^1 = \triangle q^1 \triangle \left(p_\nu \tilde{g}^{\nu 1} \right).$$

What is the minimum value of this product? Since p is an independent variable, we may take (for the moment) $\triangle p_j = 0$ so that

$$\triangle p^1 = \triangle \left(p_\nu \tilde{g}^{\nu 1} \right) = p_\nu \triangle \tilde{g}^{\nu 1}.$$

In order to determine $\triangle \tilde{g}^{\nu 1}$ we will argue that the variance of the distribution of the average of the metric tensor over a region of space-time is inversely proportional to the volume, V, i.e.,

$$Var \left(\frac{1}{V} \int_v \tilde{g}_{\mu\nu} d\mu d\nu \right) = \frac{k}{V}$$

In other words, we wish to show that if we are given a volume and if we consider the average values of the metric components over this volume, then these average values, which of course are stochastic, are less stochastic than the metric component values at any given venue in the volume, and that the stochasticity, which we can represent by the variances of the distributions of the metric components, is inversely proportional to the volume. This allows that over macroscopic volumes, the metric tensor behaves classically (i.e. according to general relativity).

As an idealization, let's assume the distribution of each metric tensor component at any venue Θ is Gaussian,

$$f_{\tilde{g}_{\mu\nu}}(g_{\mu\nu}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{g_{\mu\nu}}{\sigma} \right)^2}.$$

Note also that if $f(y)$ is a Gaussian distribution, the scale transformation $y \longrightarrow y/m$ results in $f(y/m)$ which is Gaussian with

$$\sigma^2_{(y/m)} = \frac{\sigma^2_y}{m^2}$$

Also, we define $f_{g_{\mu\nu}}$ at $e_1(g_{\mu\nu}) \equiv f_{\Theta_1}(g_{\mu\nu})$. We now require

$$Var(f_{((\Theta_1+\Theta_2+....+\Theta_m)/m)} \equiv \sigma^2_{((\Theta_1+\Theta_2+....+\Theta_m)/m)},$$

where $f_{(\Theta)}$ is normally distributed. Now again, the convolute $f_{(\Theta_1+\Theta_2)}(g_{\mu\nu})$ is the distribution of the sum of $g_{\mu\nu}$ at Θ_1 and $g_{\mu\nu}$ at Θ_2,

$$f_{(\Theta_1+\Theta_2)} = \int_{-\infty}^{\infty} f_{\Theta_1}(g^1_{\mu\nu})f_{\Theta_2}(g^1_{\mu\nu} - g^2_{\mu\nu})dg^2_{\mu\nu},$$

where $g^1_{\mu\nu}$ is defined to be $g_{\mu\nu}$ at Θ_1. Here $f_{\Theta_1} = f_{\Theta_2}$ as there are presumed to be no masses in the neighborhood of the test particle. So that,

$$f_{(\Theta_1+\Theta_2)/2} = f_{(g_{\mu\nu}/2 at \Theta_1 + g_{\mu\nu}/2 at \Theta_2)}$$

is the distribution of the average of $g_{\mu\nu}$ at Θ_1 and $g_{\mu\nu}$ at Θ_2. $\sigma^2_{(\Theta_1+\Theta_2+....\Theta_m)}$ is easily shown from the theory of normal distributions to be,

$$\sigma^2_{(\Theta_1+\Theta_2+....+\Theta_m)} = m\sigma^2_\Theta.$$

Also, $f_{(\Theta_1+\Theta_2....+\Theta_m)}$ is normal. Hence,

$$\sigma^2_{((\Theta_1+\Theta_2+....+\Theta_m)/m)} = \frac{m\sigma^2_\Theta}{m^2} = \frac{\sigma^2_\Theta}{m},$$

or the variance is inversely proportional to the number of elements in the average, which in our case is proportional to the volume. For the case where the distribution $f_{(g_{\mu\nu})}$ is not normal, but also not 'pathological', the central limit theorem gives the same result as the normal case. Further, if the function $f_{(g_{\mu\nu})}$ is indeed not normal, the distribution $f_{((\Theta_1+\Theta_2+....+\Theta_m)/m)}$ in the limit of large m is normal,

$$f_{((\Theta_1+\Theta_2+....+\Theta_m)/m)} \longrightarrow f_{((\int_V g_{\mu\nu}d\tilde{V})/V)}.$$

In other words, over any finite region of space-time, the distribution of the average of the metric tensor over the region is Gaussian. Therefore, in so far as we do not consider particles to be point sources, we may take the metric fluctuations around the location of a particle as normally distributed for for each of the metric components $\tilde{g}_{\mu\nu}$. Note however that this does not imply that the distributions for any of the metric tensor components are the same for there is no restriction on the value of the variances σ^2. Note also that the condition of normally distributed metric components does not restrict the possible particle probability

distributions, save that they be single-valued and non-negative. This is equivalent to the easily proved statement that the functions

$$f_{(x,\alpha,\sigma)} = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{1}{2}\left(\frac{x-\alpha}{\sigma}\right)^2\right)}$$

are complete for non-negative functions.

Having established that,

$$Var\left(\frac{\Theta_1 + \Theta_2 + ... + \Theta_m}{m}\right) = \frac{\sigma_\Theta^2}{m},$$

consider again the uncertainty product $\triangle q^1 \triangle p^1 = p_\nu \triangle q^1 \triangle \tilde{g}^{\nu 1}$. $\triangle q^1$goes as the volume (volume here is $V^1$the one-dimensional volume). $\triangle \tilde{g}^{\nu 1}$ goes inversely as the volume, so that $p_\nu \triangle q^1 \triangle \tilde{g}^{\nu 1}$ is independent of the volume; i.e. as one takes $q^1$to be more localized, p^1 becomes less localized by the same amount, so that for a given covariant momentum p_j (which we might call the proper momentum), $p_\nu \triangle q^1 \triangle g^{\nu 1} =$ a constant k. If $p_\nu$is also uncertain, $p_\nu \triangle q^1 \triangle g^{\nu 1} \geq k$ or equivalently,

$$\triangle q^1 \triangle p^1 \geq k$$

which is the uncertainty principle.

V. THE WAVE FUNCTION, THE TWO-SLIT EXPERIMENT, MEASUREMENT, AND THE ARROW OF TIME

If we are to derive the results of quantum mechanics purely from characteristics of the metric tensor, we need to somehow identify the quantum mechanics wave function Ψ as some function of the metric tensor. While we can easily identify the probability density$\Psi^*\Psi$ with $\sqrt{-\|g_{\mu\nu}\|}$ it is not immediately clear how to treat Ψ by itself. The utility of Ψ is that it contains phase information. Hence using Ψ allows interference phenomena. One might think then, that our stochastic space-time approach might have considerable difficulty in producing interference. If however, we assume a particle does indeed have an associated DeBroglie wavelength (we'll attempt to explain the genesis of the DeBroglie wavelength in Part II), then the metric superposition postulate can generate interference as follows:

Again, consider the free particle in space where there is no nearby mass. this condition implies that over a region of space near the particle, the metric tensor is, on average,

Minkowskian. And again, the position probability density $P(x,t) = \sqrt{-\|g_{\mu\nu}\|}$. Now consider a two-slit experiment. Let the situation $s1$ where only one slit is open result in a metric (where the stochastic elements are averaged out) $g_{\mu\nu}^{s1}$. And the case where only slit two is open, $s2$, result in $g_{\mu\nu}^{s2}$. The case where both slits are open is then (by postulate 5) is $g_{\mu\nu}^{s3} = \frac{1}{2}\left(g_{\mu\nu}^{s1} + g_{\mu\nu}^{s2}\right)$. Once we've justified the particle's DeBroglie wavelength, this will provide for interference. Note that even if the particles are sent to the screen one-by-one, the metric tensor $g_{\mu\nu}^{s3}$ still gives the probability density of the particle landing at any position on the screen and hence still gives interference. (In most of the remainder of Part I, we assume the metric stochasticity is averaged out and so we'll omit the stochasticity tilda over $g_{\mu\nu}$.)

With interference and the wave function Ψ in mind, what more can we say about metric $g_{\mu\nu}^{s1}$? Assume a particle is traveling in, say, the x^3 direction and, of course, the x^4 direction. We might expect the metric, after averaging out the stochasticty to be the Minkowski metric, $\eta_{\mu\nu}$ save for g_{33} and g_{44},

$$
g_{\mu\nu}^{s1} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & -b \end{vmatrix},
$$

where a and b are as yet undefined functions. In order that the probability density be constant, we need $\|g_{\mu\nu}^{s1}\| = -ab$ to be constant. We'll take $a = b^{-1}$ so that $\|g_{\mu\nu}^{s1}\| = \|\eta_{\mu\nu}\| = -1$.

Now, for the moment, we'll introduce an unphysical situation. Let $a = e^{i\alpha}$ where α is some as yet unspecified function of position. Consider the following metrics,

$$
g_{\mu\nu}^{s1} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \end{vmatrix},
$$

$$
g_{\mu\nu}^{s2} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\beta} & 0 \\ 0 & 0 & 0 & -e^{-1\beta} \end{vmatrix},
$$

where α and β are some unspecified functions of position. For the metrics, $\sqrt{-\left\|g_{\mu\nu}^{s1}\right\|} = \sqrt{-\left\|g_{\mu\nu}^{s2}\right\|} = 1$, and noting that for a 4x4 matrix, $\left\|\frac{1}{2}A\right\| = \frac{1}{16}\left\|A\right\|$,

$$\sqrt{-\left\|g_{\mu\nu}^{s3}\right\|} = \sqrt{\frac{-1}{16}\left\|g_{\mu\nu}^{s1} + g_{\mu\nu}^{s2}\right\|} = \sqrt{\frac{-1}{16}\left(2 + e^{i(\alpha-\beta)} + e^{-i(\alpha-\beta)}\right)} = \frac{1}{2}Abs\left(cos\left(\alpha - \beta\right)\right).$$

This is, of course, the phenomenon of interference. The metrics $g_{\mu\nu}^{s1}, g_{\mu\nu}^{s2}$, and $g_{\mu\nu}^{s3}$ represent, for example, the two-slit experiment previously described. The analogy of the function $e^{i\alpha}$ withe the wave function Ψ is obvious. However, the use of complex functions in the metric is unphysical as the resultant line element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ would be complex. But could we reproduce the previous scheme, but with real functions? The answer is yes, but first we must briefly discuss quadratic-form matrix transformations[10]. Let,

$$X = \begin{vmatrix} dx^1 \\ dx^2 \\ dx^3 \\ dx^4 \end{vmatrix},$$

and let matrix $G = g_{\mu\nu}$. Then $X^t GX = ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where X^t is the transpose of X. Consider transformations which leave the line element ds^2 invariant. Given a transformation matrix W, we can have $X = WX'$ and $X^t GX = X'^t G'X' = (X^t(W^t)^{-1})G'(W^{-1}X)$. [Note: $(WX')^t = X'^t W'^t$.] However, $X^t GX = (X^t(W^t)^{-1})(W^t GW)(W^{-1}X)$ so that $G' = W^t GW$. In other words, the transformation W takes G into $W^t GW$.

Now in the transformed coordinates, a metric $g_{\mu\nu}^{s1} \equiv G^{s1}$ goes to $W^t G^{s1}W$. Therefore $\Psi_1^* \Psi_1 = \sqrt{-\left\|W^t G^1 W\right\|} = \sqrt{-\left\|W^t\right\|\left\|G^1\right\|\left\|W\right\|}$. And $\Psi_3^* \Psi_3 = \sqrt{-\frac{1}{16}\left\|W^t\right\|\left\|G_1 + G_2\right\|\left\|W\right\|}$.

If we can find a transformation matrix W with the properties,

(i) $\left\|W\right\| = 1$,

(ii) W is not a function of α or β,

(iii) $W^t GW$ is a matrix with only real components,

then we will again have the interference phenomenon with $g'_{\mu\nu}$ real, $\Psi_1^* \Psi_1 = \Psi_2^* \Psi = 1$, and $\Psi_3^* \Psi_3 = \frac{1}{2}Abs\left(cos\frac{\alpha-\beta}{2}\right)$. The appropriate matrix W is,

16

$$W = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{vmatrix}.$$

If, as previously,

$$G^{s1} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \end{vmatrix},$$

then,

$$W^t G^{s1} W = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\cos(\alpha) & \sin(\alpha) \\ 0 & 0 & \sin(\alpha) & \cos(\alpha) \end{vmatrix},$$

so that in order to reproduce the phenomenon of interference, the transformed metric tensor will have off-diagonal entirely real terms. The coordinates appropriate to G' are $x^{1'} = x^1$, $x^{2'} = x^2$, $x^{3'} = \frac{-i}{\sqrt{2}}x^3 + \frac{1}{\sqrt{2}}x^4$ and $x^{4'} = \frac{1}{\sqrt{2}}x^3 - \frac{i}{\sqrt{2}}x^4$, which is to say that with an appropriate coordinate transformation (which is complex), we can treat the probability distribution $\Psi^*\Psi$ in an intuitive way. In so far as differential geometry is coordinate independent, we can simply ignore that the coordinate system is complex.

Incidentally, if we look at the sub-matrix,

$$\begin{vmatrix} -\cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{vmatrix},$$

it is worth noting that this represents a *rotoreflection* transformation, that is to say a rotation accompanied by a reflection. A repeated application of this transformation is suggestive of snapshots of a torsional vibration. We'll have cause to explore torsional vibrations in Part II.

Although we've explored superposition, we have yet to provide an explanation for the measurement problem and the 'delayed choice' phenomenon in the two-slit experiment. First

we need to discuss the measurement process at the slits–and also locality and objective realism.

Consider the two slit experiment using electrons. As an electron 'passes through' a slit, its electric field must distort the electrons in atoms at the surface of the slit. As such, it is a measurement of sorts. But as the electron continues through, the slit electrons of the slit atoms return to their previous states. So the 'measurement' is not preserved. The film can be run backward and it would be a valid physical situation. For there to be a true measurement then, there must be a mechanism to 'remember' the measurement [11]– a latch or flip-flop of sorts. And that would mean the film could *not* be run backward. We regard measurement then, as a breaking of time-reversal symmetry.

As it is usually maintained, a description of entanglement requires abandonment of the concepts of objective realism and also locality (We'll deal with entanglement in Part II). Indeed, Bell's theorem requires that we must abandon at least one of the two concepts. Dropping objective reality means that a physical state isn't defined until it is measured (e.g. is the cat dead or alive?). and dropping locality means that things separated in space can influence each other instantaneously (e.g. the collapse of the wave function).

Our model, while preserving objective realism, is non-local. Further, our non-locality allows for those fluctuations to move backward in time (except where measurements forbid it), retracing their paths.

Now again consider the two-slit experiment with slits A and B, and a screen at the rear of the experiment. Further, let there be a detector at A which triggers when a particle goes through slit A.

In accord with our model, there are a number of points to be made:

1) A particle will go through only *one* slit. Which one depends on the stochastic fluctuations of the metric.

2) The metric fluctuation (pilot wave) carries frequency information (via the determinant of the metric tensor) of the particle.

3) The pilot wave goes through *both* slits.

4) The probability of a particle hitting the screen is again determined by the determinant of the metric (the differential volume element).

5) In order to explain a measurement at a slit destroying the interference pattern, we'll posit that the interference phenomena are very fragile, and any disturbing of the metric

18

fluctuations can wipe out the interference information. But, if the disturbance is not ongoing, the fluctuations, as they are able to move freely in time, can go back in time and re-establish the interference at a place where there was no disturbance.. So if, for example, the particle goes through slit B, the detector at A will continue to operate (exerting a field in the vicinity of the slit) and the interference cannot re-establish. In the case where the particle goes through slit A, once detected, the detector can be switched off. But in this case the interference cannot re-establish since the disturbance cannot propagate backwards through a measurement (a flip-flop).

6) As the model has fluctuations being propagated backward in time, the delayed choice experiment follows the same arguments as the above.

Our non-locality then, requires access to the past (at least for small metric fluctuations) and so raises questions as to the arrow of time. The arrow of time' seems to emerge from statistical mechanics (via entropy). And statistical mechanics differs from mechanics in that there are many particles in play, and the particles interact. So it may well be that the arrow of times is a result of particle interactions.

For an isolated particle though, there's maybe no arrow of time, or more likely a very small arrow resultant from the slight time-reversal symmetry breaking in the weak interactions. Indeed, if there were no slight bias for an arrow of time, the universe as a whole wouldn't display one. When a large ensemble of particles interact, the arrow likely grows longer. (Whilst a particle can be run backward in time, breaking an egg can't.) Further, in the macro-world, everything is a measurement of sorts (viewing a scene gives an estimate of positions, etc.) and hence we can't run macro-world scenes backwards; a strong arrow of time has been established.

While the model is a mechanical description, it is based on an underlying stochasticity of spaced-time. So it seems that God does indeed play dice.

Part II

Crypto-stochastic Space-time

There are (at least) two problems with the stochastic space-time model: First, there's just so far one can take stochasticity. Almost by definition, it is difficult to derive deterministic equations from stochastic elements. The second and more serious problem is that our model posits a stochastic metric tensor while requiring that the determinant of the metric be non-stochastic. the square root of minus the determinant is identified with a well-defined probability density. While mathematically it is easy to get a non-stochastic determinant from a matrix with stochastic elements, it's difficult to justify with physics.

But we don't actually require stochasticity in the metric elements; we just require that they *appear* stochastic–in the sense that repeated measurements give unpredictable results. One way of obtaining this is to replace the stochasticity with an immeasurably high frequency fluctuation in the elements. The idea is that the stochastic energy fluctuations in the vacuum drive the space-time into a collective oscillatory mode (a kind of stochastic resonance). This assumption will allow us to illuminate polarization phenomena and even entanglement. Quantum mechanics then, with this modification, is deterministic but with aspects that are unmeasurable to arbitrary accuracy. Determinant but not measurable (along with non-linearity) is the book characterization of chaos. We'll make use of this in Part III.

The question is: what is the nature of these oscillations. First we'll see what oscillator models can best elucidate troublesome quantum phenomena, e.g. entanglement and optical polarization. And our descriptions must preserve objective reality (a particularly difficult problem with polarization and entanglement).

We require the metric oscillations to be of a very high frequency, sufficiently high that we can't measure them. It seems reasonable to restrict the frequency to below $10^{43}hz$ (which is the frequency where the wavelength is the Planck length) and above $10^{30}hz$ (the frequency of the highest indirectly measured gamma rays).

Taking as a hint, the roto-reflections mentioned earlier, we'll posit that the oscillations are torsional around particles (including photons).
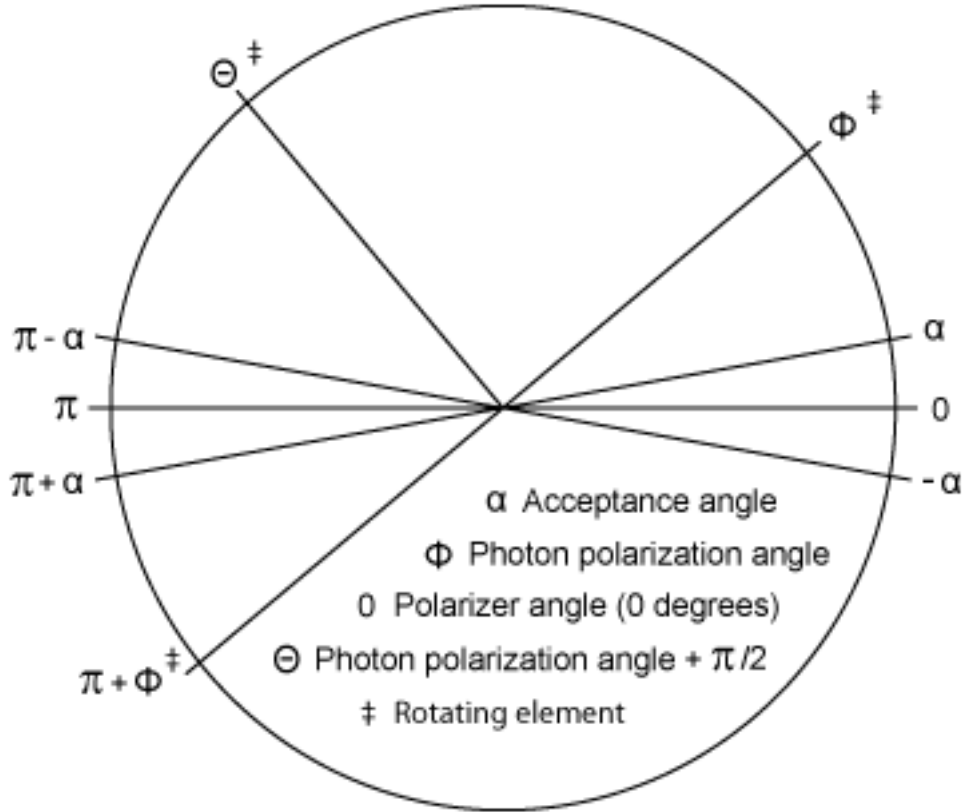
Fig. 4. A model for optical polarization.

VI. LINEAR POLARIZATION

Consider now optical linear polarization. We'll address three issues: 1-the reason half the incident photons go through a polarizer, rather than just photons with polarization oriented in the same direction as the polarizer's polarization angle (quantum mechanics has a tortuous explanation); 2-Malus's Law; and 3-the situation when a third polarizer is inserted between a pair of crossed polarizers.

Consider Figure 4. Assume a polarizer (in the x, y coordinates perpendicular to z, the direction the photon moves) with polarization orientation along the $0 - \pi$ axis. And consider a photon with polarization angle Φ encountering the polarizer. Our torsionally oscillating space-time model assumes the photon is essentially oscillating through $\varphi\pi$ radians around a point $\pi/2$ radians from the polarization angle. So in figure 4, the photon is oscillating around Θ from Φ to $\pi + \Phi$. (As a short-hand, rather than speaking of the space-time torsionally oscillating and carrying the photon with it, we'll refer to it simply as the oscillating photon.)

A frictionless torsional spring is governed by the equation, $\theta = k * cos(\omega t + \Phi)$ where ω is

21

the oscillation frequency and k is a constant (for the moment) involving the torsional spring stiffness and the angle through which the spring oscillates. At $t = 0$, θ is at the extremum, Φ.

A photon with a well-defined polarization direction (here Φ) oscillates as it reaches the polarizer. It is clear that the more time the rotating photon's polarization vector lies within the acceptance angles, the more likely the photon will pass through the polarizer. And so the angular velocity is proportional to the likelihood of the photon *not* getting through (the attenuation A). So A_θ (at θ) is proportional to the angular velocity, $d\theta/dt = -k\omega * sin(\omega t + \Phi)$. Or referenced to φ (which is $\theta + \pi/2$) $d\varphi/dt = k\omega * cos(\omega t + \Phi + \pi/2)$. Or $A_\theta = k_1 d\varphi/dt$ where k_1 is a constant.

The maximum throughput is at Φ which is at $\pi/2$ from Θ, and θ is always equal to $\varphi + \pi/2$. So as the attenuation follows a cosine law, so to does the intensity, I. We see that the transmission increases as the cosine of the angle between the photon polarization angle and the center of the acceptance angle. I.e. the intensity I_Θ at angle Θ is, $I_\Theta = k_2 d\varphi/dt$ where k_2 is a constant.

There are three obvious problems: First the transmission amplitudes are small. With a perfect polarizer; the acceptance angle tends to a delta function and the transmission become infinitesimal. Second, the minimum transmission isn't equal to zero. And third, the functional form of the intensity is wrong; it should go not as cosine, but as cosine squared. These problems can be handled by considering not only oscillations in x and y, but also in z and t. Relativity ideas suggest oscillations in all coordinates.

Consider oscillations in the two directions perpendicular to the coordinates of figure 4, namely t and z. The photons travel along z en route to the polarizer. If there are oscillations in t against z, the world-line of a photon is not the light cone, a 45 degree line in a Minkowski diagram, but a wavy line as in figure 5. So, at any time t, the photon exists at a linear series of values of z giving the photon some of the attributes of a 'string' (or dotted line) of well-defined length.

We assume that the t z oscillation is synchronized with the x y oscillation and that the zero point is $\pi/2$ displaced from the axis of rotation just as in the xy case. Now consider a photon with, for example, polarization angle=0 encountering a polarizer with polarization angle also equal to zero. If when it reaches the polarizer, the oscillating photon's rotation lines up with the polarizer's axis (very rarely), it goes through. If not, then the photon is

displaced slightly back in time and with a slightly lower angle. Again, it goes through, or not. If not, again the photon slightly back in tame and angle encounters the polarizer. The process continues until either the photon goes through the polarizer or the time oscillation goes forward again. So, at this point, the transmission probability is $1/2$. But if the photon has not gone through, the photon will have rotated through π radians and the argument applies again. This is so because a photon can go through with polarization 0 or π. We get another probability of $1/2$ giving a transmission probability of unity. Now, using the same argument as with the xy case, if the photon polarization is at an angle ϕ with respect to the polarizer angle, then the probability of transmission due to the oscillations goes as the cosine of the angle. So considering both time and space oscillations gives an attenuation of cosine squared of the angle. And that is the expected result, i.e. Malus's law. When the photon enters the polarizer, it is 'prepared' by the polarizer. That is to say that since the polarizer can admit only photons with polarization direction the same as that of the polarizer, the photon is forced to the polarization of the polarizer. The photon continues to rotate, but (usually) around a different angle.

In the case of two crossed polarizers, there is nothing new; no light gets through. But if a third polarizer at, say, a 45 degree angle is interposed between the two crossed polarizers, $1/4$ of the light gets through. The conventional quantum mechanics explanation is that when the photon encounters the interposed polarizer, it decomposes into components parallel and perpendicular to the polarization angle of the interposed polarizer, and then either gets transmitted or absorbed with a probability based on the amplitudes of the decomposed polarization vector.

Our model explains this effect as follows: when the oscillating photon encounters the interposed polarizer, it, as described earlier, goes through with some probability. But as it does so, it is 'prepared' (as described above) to have the same polarization angle as the interposed polarizer. So now the newly prepared photon has a polarization angle that is no longer at a right angle to the third polarizer. So there is a probability of the photon getting through the third polarizer.

Using the word 'probability' might seem to imply that the transmission is, in the quantum mechanical sense, probabilistic. But it is actually deterministic in principle. If we knew the rotational angle of the photon when it encountered the polarizer, we would know if the photon would or would not get through. In practice though, since the rotational frequency
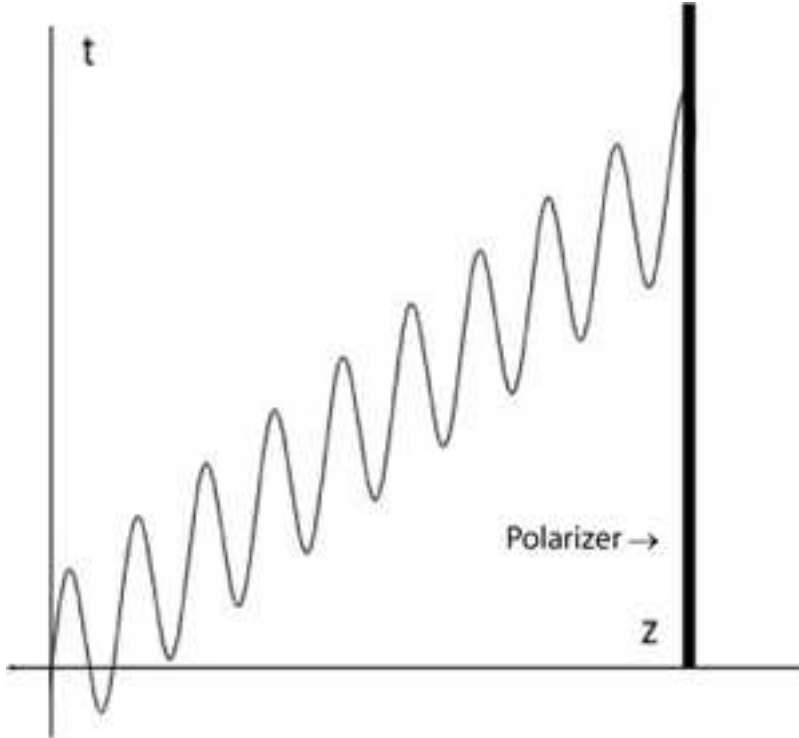
Fig 5. Oscillations of t against z

as photon approaches polarizer.

is way too high to measure, in practice, all we can give is a probability.

VII.   A NOTE ON ENTANGLEMENT

As our model seems to give a model of polarization, one might ask if the model has anything to say about entanglement. It might. Usually, a pair of entangled particles emerge from a single venue. One might have that (in our model) the particles are locked to exactly the same oscillation behavior (or perhaps are locked π radians out of phase). And when they separate, they remain so locked. In the language of quantum mechanics, they share a common wave function. When one of the particles is measured, its oscillation freezes at its current oscillation angle. Again, in the language of quantum mechanics, the wave function collapses. In our language, the metric distortion dissipates and the other particle also freezes at the same angle (or the same angle $+\pi$). This model also addresses single particle spin up/down measurements in Stern-Gerlach experiments (where the angle of the measurement apparatus is varied). Note that this model is, of course, non-local, as required by Bell's theorem [19]. And finally, as the model incorporates objective reality, Schrödinger's cat is

24

either alive or dead but not both.

In this crypto-stochastic model, after stochastic resonance creates the oscillations, the model is completely deterministic. So God does play dice, but only to set things going.

Part III

Chaotic Space-time (soon to come)

The introduction of stochastic space-time admits a phenomenological explanation of some elements of quantum mechanics. The extension to crypto-stochastic space-time yields possible explanations of more quantum phenomena. The aim is to provide a mechanical system to model all the elements of quantum mechanics. There is much left to do. The principle task is to explain the origins of the space-time oscillations, and to provide 'field equations' as in general relativity, to give quantitative results (from which, the Schrödinger equation, among others, will drop out).

The first thing to note is that the present model, derived as it is from ideas of General Relativity, is inherently non-linear, although as the constant of gravitation is very small (compared to the electro-weak force) the non-linear effects are exceedingly small. Further, the model is (once the oscillations are established) completely deterministic. Further, though deterministic, the state (phase angles) of the oscillations is unmeasurable. But these are the defining elements of a chaotic system: non-linearity, deterministic, immeasurable. We will demonstrate (in a forthcoming paper) that do to stochastic resonance, the stochastic space-time has large-scale periodicity. And this deterministic periodicity allows chaos which then allows small-scale, self-organizing periodicity at the scale of the elementary particles. We use the techniques of chaos theory to obtain, if not the quantum mechanics analogy of the relativity field equations, the behavior of space-time at small dimensions.

And so, perhaps Einstein was right after all; God does not play dice–or at the most, exceedingly rarely.

Acknowledgments

I should like to thank Drs. Norman Witriol and Hans Fleischmann, as well as Nick Taylor for helpful conversations and ideas.

---

[1] C. Frederick, Phys. Rev. D **13**, 12, 3183 (1976)

[2] B. Schweizer and A. Sklar, Pac. J. Math **10**, 313 (1960)

[3] A. March, Z. Phys. **104**, 93 (1934)

[4] A. March, Z. Phys. **105**, 620 (1937)

[5] D. Blokhintsev, Fiz. Elem. Chastits At. Yad. **5**, 606 (1975)

[6] C. Masreliez, *www.estfound.org/quantum.htm*

[7] N. Cobern, *Vector and Tensor Analysis* (MacMillan, London, 1955)

[8] W. Rosser, *An Introduction to the Theory of Relativity* (Butterworths, London, 1964)

[9] P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, N.J. 1942)

[10] F. Hohn, *Elementary matrix Algebra* (MacMillan, London, 1958)

[11] C. Frederick, *We are the Cat* Asimov's Magazine (September 2006)

[12] C. Frederick, *The Trojan Carousel* (Kindle books, 2011)

[13] D. Bohm, Phys Rev. **85**, 166-193 (1985)

[14] H. Nikolic, Found Phys Sett. **18**, 549 (2004)

[15] L. Rozema et. al, Phys. Rev. Sett. **109**, (2012)

[16] O. Oreshkov, T. Brun, Phys Rev. Sett. **95**, (2005)

[17] Y. Aharonov, L. Vaidman, Phys. Rev. A **41**, 11 (1990)

[18] Y. Aharonov, D. Albert, L. Vaidman, Phys Rev. Sett. **60**, (1988)

[19] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge 2004)